Calculus Warm Up #9-5

Random Group Test Practice:

Start by going over last night's homework.

21. $y = \frac{1}{b^2}\left[\ln(a+bx) + \frac{a}{\cancel{a+bx}}\right]$ $a(a+bx)^{-1}$

$y' = \frac{1}{b^2}\left[\frac{b}{a+bx} - \frac{a(a+bx)^{-2}(b)}{1}\right]$

$= \frac{1}{b}\left[\frac{(a+bx)}{(a+bx)^2} \quad \frac{a}{(a+bx)^2}\right]$

$= \frac{1}{b} \cdot \frac{a+bx-a}{(a+bx)^2}$

$= \frac{\cancel{b}x}{\cancel{b}(a+ax)^2}$

23. $y = -\frac{1}{a}\ln\left(\frac{a+bx}{x}\right)$

$u = \frac{a+bx}{x}$

$\frac{du}{dx} = \frac{x(b)-(a+bx)(1)}{x^2}$

$\frac{du}{dx} = \frac{bx - bx - a}{x^2}$

$\frac{du}{dx} = -\frac{a}{x^2}$

$y' = -\frac{1}{\cancel{a}}\left(\frac{\cancel{x}}{a+bx}\right)\left(-\frac{\cancel{a}}{x^{\cancel{2}}}\right)$

$y' = \frac{1}{x(a+bx)}$

$\frac{d}{dx}[\ln u] = \frac{1}{u}\cdot\frac{du}{dx}$

25. $y = \ln(e^{-x^2})$

$y = -x^2$

$y' = -2x$

27. $y = x^2e^x$

In Exercises 85–92, use L'Hôpital's Rule to evaluate the given limit.

85. $\lim_{x\to 1}\frac{(\ln x)^2}{x-1}$

87. $\lim_{x\to\infty}\frac{e^{2x}}{x^2}$

89. $\lim_{x\to\infty}(\ln x)^{2/x} = \infty^0$

91. $\lim_{n\to\infty}1000\left(1+\frac{0.09}{n}\right)^n$

let $y = \lim_{x\to\infty}(\ln x)^{2/x}$

$\ln y = \lim_{x\to\infty}\frac{2}{x}\ln(\ln x) = \frac{2}{\infty}\cdot\ln(\infty)$

$\ln y = \lim_{x\to\infty}\frac{\frac{2}{x\ln x}}{1} \quad = \frac{\infty}{\infty}$

$= \frac{2}{\infty\cdot\infty}$

$= 0$

$\ln y = 0$

$e^0 = y$

$y = 1$

1. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \frac{x-2}{x^2+1}$?

2. If $a \neq 0$ then $\lim_{x \to a} \frac{x-a}{x^2-a^2} =$

Answers

1) $\frac{-x^2+4x+1}{(x^2+1)^2}$

@ $x = 2 \rightarrow \boxed{\frac{1}{5}}$

$\boxed{2)\ \frac{1}{2a}}$

3. When is the graph of $y = 3x^4 - x^3 + 5x - 2$ concave down?

4. Determine the value of m that makes the following function continuous

$$f(x) = \begin{cases} mx + 2 & for \quad 0 \le x \le 3 \\ x^2 + 4 & for \quad 3 < x \le 4 \end{cases}$$

3) concave down on $(0, \frac{1}{6})$

4) $m = \frac{11}{3}$

5. Given $y = 2 + xy$, find all points (x, y) on the curve where the tangent line has a slope of ½.

$$\boxed{\begin{array}{l} 5)\ (3,-1)\ \&\ (-1,1) \\ 6)\ (2x+2)e^{x^2+2x} \end{array}}$$

6. Find y' given $y = e^{x^2+2x}$

$$y - xy = 2$$
$$y(1-x) = 2$$
$$\boxed{y = \frac{2}{1-x}}$$
$$y = 2(1-x)^{-1}$$
$$y' = -2(1-x)^{-2}(-1)$$
$$y' = \frac{2}{(1-x)^2}$$

$$\frac{2}{(1-x)^2} = \frac{1}{2}$$
$$(1-x)^2 = 4$$
$$1-x = \pm 2$$
$$1 \pm 2 = x$$
$$x = 3, -1$$

7. Find the derivative of $f(x) = \ln \dfrac{\sqrt{2x-1}}{x^2(x+2)^{\frac{1}{3}}}$

$$\boxed{f'(x) = \frac{1}{2x-1} - \frac{2}{x} - \frac{1}{3(x+2)}}$$

or

$$f'(x) = \frac{-11x^2 - 11x + 12}{3x(x+2)(2x-1)}$$

8. Find the derivative of $y = (x^2)^{2x}$

$$\boxed{4x^{4x}(1 + \ln x)}$$

$$y = x^{4x}$$

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9. Find the following limit: $\boxed{\lim_{x\to1^+}(x-1)^{\ln(x)}}$ $= (1-1)^{\ln 1} = 0^0$ indeterminate.

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10. Find the following limit: $\lim_{x\to1}\left(\dfrac{x}{x^2-1}-\dfrac{1}{x-1}\right) = -\infty$

let $y = \lim_{x\to1^+}(x-1)^{\ln x}$

$$\ln y = \lim_{x\to1^+}\left[\ln\left((x-1)^{\ln x}\right)\right]$$

$$\boxed{\lim_{x\to1^+}(x-1)^{\ln x} = 1}$$

$$\ln y = \lim_{x\to1^+}\left[\ln x\left(\ln(x-1)\right)\right]$$

$$\ln y = \lim_{x\to1^+}\left[\frac{\ln(x-1)}{\frac{1}{\ln x}}\right] = \frac{\ln(1-1)}{\frac{1}{\ln 1}} = \frac{-\infty}{\infty}$$

* Now L'Hôpital's Rule

$$\ln y = \lim_{x\to1^+}\left[\frac{\frac{1}{x-1}}{\frac{-1}{x(\ln x)^2}}\right]$$

$$\ln y = \lim_{x\to1^+}\left[\frac{1}{x-1}\cdot\frac{x(\ln x)^2}{-1}\right]$$

$$\ln y = \lim_{x\to1^+}\left[\frac{-x(\ln x)^2}{x-1}\right] = \frac{-1(\ln 1)^2}{1-1} = \frac{0}{0}$$

$$\ln y = \lim_{x\to1^+}\left[\frac{-\ln x(2+\ln x)}{1}\right] = -\ln 1(2+\ln 1) = 0$$

$$\ln y = 0$$

$$e^0 = y \longrightarrow y = 1$$

9. Find the following limit: $\boxed{\lim_{x\to1^+}(x-1)^{\ln(x)}}$

10. Find the following limit: $\lim_{x\to1}\left(\dfrac{x}{x^2-1}-\dfrac{1}{x-1}\right) = \dfrac{1}{1-1} - \dfrac{1}{1-1}$

$$= \frac{1}{0} - \frac{1}{0}$$

$$\infty - \infty \quad \text{indeterminate}$$

$$\lim_{x\to1}\frac{x-(x+1)}{x^2-1}$$

$$\lim_{x\to1}\frac{-1}{x^2-1}$$

$$\frac{-1}{1-1}$$

$$-\frac{1}{0} \longrightarrow \boxed{-\infty}$$

HW: More review!

Answers posted by mid-day Saturday